$\boxed{1}$

$C: y = x^2 - 4x + 3 = (x-1)(x-3)$

$l: y = mx - m = m(x-1)$

(1) $x^2 - 4x + 3 = mx - m$

$x^2 - (m+4)x + m + 3 = 0$

判別式をDとすると

$D = (m+4)^2 - 4(m+3)$

$\quad = m^2 + 4m + 4 = 0$

$(m+2)^2 = 0$

$m = -2$

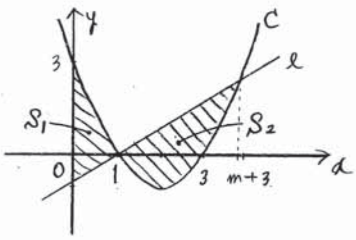
$\therefore \underline{m_0 = -2}$ //

(2) $x^2 - (m+4)x + m + 3 = 0$

$(x-1)(x-m-3) = 0$

$x = 1,\ m+3$

$m > m_0 = -2$ より. $m+3 > 1$



よって.

$$S_1 = \int_0^1 \{(x^2 - 4x + 3) - (mx - m)\}dx$$

$$= \int_0^1 \{x^2 - (m+4)x + m + 3\}dx$$

$$= \left[\frac{1}{3}x^3 - \frac{m+4}{2}x^2 + (m+3)x\right]_0^1$$

$$= \underline{\frac{1}{2}m + \frac{4}{3}} \text{ //}$$

$$S_2 = \int_1^{m+3} \{(mx - m) - (x^2 - 4x + 3)\}dx$$

$$= -\int_1^{m+3} (x-1)\{x - (m+3)\}dx$$

$$= -\left\{-\frac{1}{6}(m+3-1)^3\right\}$$

$$= \underline{\frac{1}{6}(m+2)^3} \text{ //}$$

(3). $S_2 - 2S_1 = f(m)$ とおくと

$$f(m) = \frac{1}{6}(m+2)^3 - 2\cdot\left(\frac{1}{2}m + \frac{4}{3}\right)$$

$$= \frac{1}{6}m^3 + m^2 + m - \frac{4}{3}$$

$$f'(m) = \frac{1}{2}m^2 + 2m + 1$$

$$= \frac{1}{2}(m^2 + 4m + 2)$$

$f'(m) = 0$ のとき. $m = -2 \pm \sqrt{2}$

$m > -2$ における増減表をかくと.

m	-2	$\cdots$	$-2+\sqrt{2}$	$\cdots$
$f'(m)$		$-$	0	$+$
$f(m)$		↘		↗

$$f(-2+\sqrt{2}) = \frac{1}{6}(-2+\sqrt{2}+2)^3 - (-2+\sqrt{2}) - \frac{8}{3}$$

$$= -\frac{2}{3}(\sqrt{2}+1)$$

$\therefore$ $\underline{m = -2+\sqrt{2} \text{ のとき. 最小値 } -\frac{2}{3}(\sqrt{2}+1)}$ //

$\boxed{2}$

(1) $y=|x^2-2x-3|=|(x+1)(x-3)|$

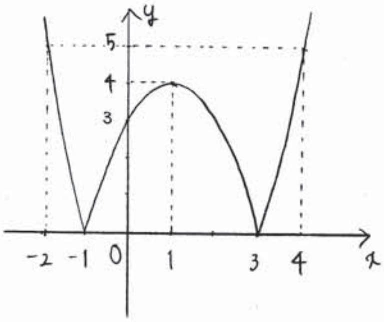
$x \leqq -1,\ 3 \leqq x$ のとき

$y=x^2-2x-3=(x-1)^2-4$

$-1<x<3$ のとき

$y=-(x^2-2x-3)=-(x-1)^2+4$

よって、グラフは



(2) $|x^2-2x-3|=a$ の実数解の個数は

$y=|x^2-2x-3|$ と $y=a$ の共有点の個数と

等しい。よって、(1)のグラフより

$a<0$ のとき　0個

$a=0,\ a>4$ のとき　2個

$a=4$ のとき　3個

$0<a<4$ のとき　4個 //

(3) $||x^2-2x-3|-6|=2$

$|x^2-2x-3|-6=\pm 2$

$|x^2-2x-3|=8,\ 4$

(2)より　$a=8$ のとき　2個

$a=4$ のとき　3個

よって、5個 //

$\boxed{3}$

(1) $0^\circ < \theta < 90^\circ$ より $\sin\theta > 0$ なので、

$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{3}{5}} = \sqrt{\frac{2}{5}}$$

$$\sin 2\theta = 2\sin\theta\cos\theta = 2\cdot\sqrt{\frac{2}{5}}\cdot\sqrt{\frac{3}{5}} = \frac{2\sqrt{6}}{5}$$

よって

$$a = \frac{2\sqrt{5}\left(\sqrt{\frac{2}{5}}+\sqrt{\frac{3}{5}}\right)-5\cdot\frac{2\sqrt{6}}{5}}{2}$$

$$= \underline{\sqrt{2}+\sqrt{3}-\sqrt{6}} \;/\!/$$

(2) $$\frac{1}{a} = \frac{1}{\sqrt{2}+\sqrt{3}-\sqrt{6}}$$

$$= \frac{\sqrt{2}+\sqrt{3}+\sqrt{6}}{(\sqrt{2}+\sqrt{3}-\sqrt{6})(\sqrt{2}+\sqrt{3}+\sqrt{6})}$$

$$= \frac{\sqrt{2}+\sqrt{3}+\sqrt{6}}{(\sqrt{2}+\sqrt{3})^2-(\sqrt{6})^2}$$

$$= \frac{\sqrt{2}+\sqrt{3}+\sqrt{6}}{2\sqrt{6}-1}$$

$$= \frac{(\sqrt{2}+\sqrt{3}+\sqrt{6})(2\sqrt{6}+1)}{(2\sqrt{6}-1)(2\sqrt{6}+1)}$$

$$= \underline{\frac{7\sqrt{2}+5\sqrt{3}+\sqrt{6}+12}{23}} \;/\!/$$